Stochastic Pricing

Southeastern Actuaries Conference

Cheryl Angstadt

November 15, 2007
Agenda

- Background
- Drivers
- Case Study
- PBA and SOS
- Approaches
Background
What do we mean by stochastic pricing?

*Modeling of outcomes under a large number of randomly-generated future experience scenarios*

- The scenario generator should be consistent with statistical distribution of possible values for stochastically-modeled variables.
- Variables modeled stochastically can be limited to one (e.g., interest rates) or several (e.g., interest rates, equity returns, and mortality).
- Some secondary effects, e.g., policyholder behavior.
Deterministic versus Stochastic

- Deterministic modeling derives outcomes under a finite set of fully-defined scenarios
  - e.g., “New York Seven” interest rate scenarios
  - Scenarios and outcomes don’t have associated probability weightings

- Stochastic modeling derives the statistical distribution of possible outcomes
  - Facilitates the quantification of risk/return trade-off
  - Typically uses a large number of scenarios; eliminates the chance that the deterministic approach omits a significant scenario
  - Equal weighting of scenarios
  - Representative scenarios with unequal weights
Certain policyholder behaviors must be modeled to evaluate the volatility of results in the tails

- Policyholder behavior due to variations in economic scenarios can affect the results of stochastic modeling just as dramatically as the choices or standards used to create a scenario set.

- Many of these variations in policyholder behavior are often modeled insufficiently, if at all.
  - Dynamically adjust premium persistency
  - Dynamic withdrawals and partial withdrawals
  - Guarantee resets
  - Transfers between variable and fixed subaccounts

- Modeling key policyholder behaviors can assist writers in assessing the risks of the guaranteed benefits and demonstrate the impact of policyholder behaviors on the level of required capital.
Most companies do not make an adjustment for the cost of interest rate embedded options in assets and liabilities (other than EIA).

Methodology for Reflecting Interest Rate Embedded Options in Assets and Liabilities (Percent of Responses)*

- UL NLG: 61% None, 34% Mean of Scenarios, 5% "Haircut" to Interest Margin
- UL: 74% None, 21% Mean of Scenarios, 6% "Haircut" to Interest Margin
- FA: 44% None, 41% Mean of Scenarios, 14% "Haircut" to Interest Margin
- EIA: 20% None, 69% Mean of Scenarios, 11% "Haircut" to Interest Margin

* Source: 2006 Tillinghast Pricing Methodology Survey

Relative to the prior survey, the proportion of companies making provision for embedded options in pricing has increased, but only slightly.
Drivers
A number of factors are leading to an increasing level of interest in stochastic pricing . . .

- Management information demands
  - Desire to understand distribution of possible results, not just expected results
  - Stochastic techniques are an integral part of many risk management programs

- Falling equity market
  - Benefits are in-the-money; stochastic modeling can be used to value liability options

- Low interest rates
  - Credited rates have fallen close to guaranteed levels in many cases, leading to spread compression; stochastic modeling can help quantify the impact of further falls in interest rates

- High interest rates
  - Disintermediation risk

Continued…
A number of factors are leading to an increasing level of interest in stochastic pricing...

- Regulatory
  - RBC C3 Phase 1
  - RBC C3 Phase 2
  - RBC C3 Phase 3 - PBA
  - AG39
  - International Accounting Standards may necessitate the use of stochastic techniques to value options and guarantees

- Rating agencies have been willing to consider lowering capital requirements based on results of stochastic testing

Continued…
A number of factors are leading to an increasing level of interest in stochastic pricing . . .

- Economic capital
  - Companies want to reflect true cost of capital
- Required to assess
  - Value of options and guarantees
  - Extent of non-diversifiable risk
  - Cost of unhedged risk
- Technology
  - Improvements in processing speed are making stochastic pricing more feasible
  - Software platforms are evolving
Case Study
Universal Life product with Secondary Guarantees
Product description

- Male Preferred Nonsmoker, Issue Age 65
  - Level pay premium $24
  - Single pay premium $240
- Face amount $2,000,000
- Premium paid is the amount required to satisfy the SG requirement
- Shadow fund design
Universal Life product with Secondary Guarantees
Statutory reserve methodology

- Statutory reserve under PBA
  - Gross Premium Valuation
  - Max (Deterministic Reserve, Stochastic Reserve)
  - Deterministic Reserve
    - Max (seriatim reserve, cash value)
    - ‘Best estimate’ assumptions include margin
  - Stochastic Reserve
    - Reserves: CTE 65
    - Total asset requirement: CTE 90
    - Greatest present value of accumulated deficiencies
Universal Life product with Secondary Guarantees
Assumptions and Margins

- Mortality
  - Company experience
  - 10% margin

- Lapse
  - Company and industry experience
  - Dynamically adjusted AV is zero or negative, and shadow fund is positive
    - Zero when no future premiums due
    - 50% when future premiums are required
  - 50% margin (reduction to baseline rates)

Continued…
Universal Life product with Secondary Guarantees
Assumptions and Margins

- Premium Pattern
  - PBA specifies 4 premium scenarios
    - Minimum premium
    - No future premiums
    - Pre-payment single premium
    - Pre-payment level premium
  - Company experience
    - 75% Single Premium
    - 25% Level Premium

Continued…
Universal Life product with Secondary Guarantees
Assumptions and Margins

- Expenses
  - Company experience
  - No margin since impact assumed to be immaterial

- Assets
  - Non-callable bonds average maturity of 12.8 years
  - Default costs based on company experience

- Interest Rates
  - Deterministic scenario is prescribed
  - Stochastic
    — TAS CAP:Link
    — C3 P1 pre-packaged
UL SG Statutory Reserves by duration

Reserve Stream Comparison

- AXXX Reserve
- PBA Reported Reserve
- ULCRVM Reserve

Reserve Per 1,000 Face Amount

© 2007 Towers Perrin
ULSG profit testing shows considerable variability in the stochastic results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>ROI</th>
<th>Profit Margin at 6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>11.5%</td>
<td>5.8%</td>
</tr>
<tr>
<td>Stochastic:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>17.7%</td>
<td>23.6%</td>
</tr>
<tr>
<td>95th</td>
<td>15.9%</td>
<td></td>
</tr>
<tr>
<td>75th</td>
<td>13.4%</td>
<td>9.4</td>
</tr>
<tr>
<td>Median</td>
<td>11.2%</td>
<td>6.1</td>
</tr>
<tr>
<td>25th</td>
<td>9.0%</td>
<td>2.6</td>
</tr>
<tr>
<td>5th</td>
<td>4.5%</td>
<td>(1.1)</td>
</tr>
<tr>
<td>Min</td>
<td>(7.5)%</td>
<td>(5.2)</td>
</tr>
<tr>
<td>Average</td>
<td>10.7%</td>
<td>6.1</td>
</tr>
<tr>
<td>Deterministic – RBC</td>
<td>11.9%</td>
<td>5.7%</td>
</tr>
</tbody>
</table>
PBA and SOS
A principles-based approach captures all material financial risks, benefits and guarantees associated with the contracts

- Utilizes risk analysis and management techniques to quantify the risk
- Permits the use of credible company experience to establish assumptions for risks over which the company has some control
- Provides for use of prudent estimate assumptions
PBA Guiding principles

- Reserves are based on prospective valuation method
- All risks must be reflected that a company considers
- A deterministic approach may be sufficient for some products while stochastic may be required for others
- For risks that a company can control, assumptions should reflect blend of experience and prescribed assumptions
  - Prescribed assumptions/methods for other risks

Continued…
PBA Guiding principles

- Assumptions not stochastically modeled should be based on prudent estimates
- Assumptions are not locked in at issue
- Actuary must consider the model’s limitations when
  - Setting assumptions and determining appropriateness of resulting reserve levels

The use of assumptions and risk management strategies should not be constructed to reduce reserves without reducing risk
PBA Reserves and Capital

- Gross Premium Valuation
  - Prudent Estimate Assumptions

- Deterministic
  - Single scenario approach
  - Seriatim
  - Cash value floor

- Stochastic
  - Captures material tail risks
  - Aggregate calculation
  - Greatest PV of accumulated deficiencies

*Pricing under PBA may require stochastic-on-stochastic projections to determine reserve and capital levels over the life of the policy*
Stochastic-on-stochastic pricing

Model Point 1, Scenario 1
30-year projection

Model Point 1, Scenario 2
30-year projection

Model Point 100, Scenario 100
30-year projection

ILLUSTRATIVE
Projecting 1000+ scenarios presents numerous modeling challenges

Model speed is a key consideration

- What strategies can be employed to improve model speed and efficiency?
- Can we reduce the number of scenarios required without compromising the integrity of results?
- What other techniques can be employed?
Approaches
A range of approaches is available when performing stochastic pricing

- Stop selling new business
- Ignore new requirements – assume in-force sufficiencies offset any additional required capital
- Determine a ‘range’ of profitability results
- Develop a grid of scenario paths based on specified criteria
- Perform limited Stochastic-on-Stochastic (“SOS”)
- Complete full SOS
- Improve technology
- Apply scenario reduction techniques
Stop selling new business

- Seems extreme

- Advantage
  - Simple to implement
  - Inexpensive – no new staff, hardware, etc., required

- Disadvantage
  - Reduces market share
Ignore the new requirements

- Assume sufficiencies of the in force block will fund any capital and reserve deficiencies of new blocks

- Advantages
  - Simple to implement
  - Inexpensive – no new staff, hardware, etc., required
  - Less extreme than previous solution

- Disadvantage
  - Inconsistent with valuation projections
    — But perhaps not needed
Adopt a ‘range’ approach for presenting summary of profit results

- Approach
  - Project 1,000 scenarios at MSD only with no SOS projections
  - Determine Best Estimate, Break even, and Optimistic results

- Advantages
  - Simple enough that management can understand
  - Avoid the use of SOS

- Disadvantages
  - Ignores future capital requirements
Develop a grid of factors to apply based on a product characteristic/ITM

- Vary by level of “in-the-moneyness”
- Grid contains factors that would be applied to some metric used to estimate capital requirements, e.g., NAR
- Factors must be calibrated
Limited “stochastic-on-stochastic” projections

- PBA pricing requires recalculation of reserves and capital at specified points in time, e.g., annually
  - Very computation intensive
- Limited SOS, also known as “stochastic-on-deterministic”
  - Calculations occur less frequently than annually, e.g., every five years
    - Interpolate in interim years
- Deterministic-on-Stochastic
  - Small number of “outside” scenarios
  - 1000 “inside”
Limited “stochastic-on-stochastic” projections

Advantages
- Reduced runtime
- Retain reasonable approach for capital and reserves

Disadvantages
- Less accurate than full SOS
- May need to prove consistency with full SOS
Perform full “stochastic-on-stochastic” projections

- Stochastic projections performed annually throughout projection

- Advantages
  - Accurate and consistent with PBA regulations

- Disadvantages
  - Significant increase in runtime
Technology improvements

- Platform overhaul
  - Need software platform tailor-made for intensive computations

- Grid computing
  - Throw hardware at the problem
  - Interim solution until next generation software available

- More efficient model coding
  - Implement closed-form solutions for calculations using fixed assumptions, e.g., GMP, SG SP, etc.
  - Annual calculations in lieu of monthly beyond a designated point in time, e.g., after 10 years
  - Calculate rates when needed, not necessarily monthly, e.g., mortality rates may change only in policy month 1
Meaning of modeling efficiency – more than being efficient with regard to scenarios

- Scenario design
  - How to minimize or reduce the number of scenarios required in a stochastic run

- Mathematical and/or model design
  - e.g., use of Black-Scholes (“closed form” solution) instead of a full-blown Monte Carlo

- Model data design
  - How to group or cluster data to reduce the number of model points required

- Hardware design

- Software design
Promising research identified so far – all in the scenario design area

- Probabilities are assigned to each representative scenario
  - For example, 200 scenarios might be assigned to NY 7
  - Probability determined by number of scenarios assigned to each representative scenario

- Variation where representative scenarios have equal weight

- Usage of the variance of CTE, and how to reduce the sampling error in the CTE values for a given number of scenarios
Simpler methods to select the worst scenarios can be employed

- Recall capital measure is based on CTE(90)
  - Implies results from 900 scenarios are not used
  - Can we just project the worst 100 and average all results?
- Use base projection
- Rank 1,000 C3 scenarios based on PV book profits (no capital)
- Select worst 100
  - By asset allocation
  - By benefit type
- Take average these 100 scenarios
- Limit projection period for C3 calculations
  - 10 years, on average
There are limitations to scenario reduction techniques

- Efforts to reduce runtime have focused on
  - Reducing the number of model points used
  - Reducing the number of scenarios processed

- Historically, actuarial research has concentrated only on the use of representative scenarios and weighting the results based on the probability associated with each representative
  - Does not adequately represent the overall behavior of all scenarios

**The goal of model efficiency is to reduce the entire processing time of the various capital and reserve models**
Predictive modeling may be used to improve scenario reduction techniques

- Predictive modeling: a process where model is created or chosen to best predict the probability of an outcome (Source: Wikipedia)

- Projection Pursuit Regression model
  - Very good job creating models not affected by outliers
  - Replicates the overall behavior of a high dimensional model
  - Very quick when evaluating additional input besides the training data

- Weightings applied to representative scenarios causes the results to be dependent upon how the weights are obtained or used

- Instead use the representative scenarios without weights as training data for the predictive model
Relative Error of PPR – CTE 65

Results very tightly bunched around the median (heavy line) with as few as 400 scenarios.
Process is liberally biased, understating both reserves and capital.

Future research:
- Examine ways to reduce bias
- Consider other predictive models
Questions?